MODEL QUESTION PAPER
ON
DIGITAL SIGNAL PROCESSING

SUBJECT TEACHER :- ROHIT JAIN BRANCH :- IT (V^{th} Sem)

Q.1 :- Explain various properties of discrete time LTI System?

Q.2. obtain the Convolution of the following sequences

$$x[n] = u[n]$$
$$h[n] = 2^n u[n]$$

Q.3. Explain various properties of Z Transform?

Q.4. By using partial fraction expansion method, find the inverse Z Transform of

$$H(z) = \frac{-4 + 8z^{-1}}{1 + 6z^{-1} + 8z^{-2}}$$

Q.5. what is sampling theorem? Explain Interpolation method for reconstruction?

Q.6 Explain Aliasing?

Q.7. Determine the DFT of the sequence

$$x(n) = \begin{cases} 1/4 , & \text{for } 0 \le n \le 2 \\ 0 , & \text{otherwise} \end{cases}$$

Q.8. Compute the DFTs of the sequence $x(n) = \cos \frac{n\pi}{2}$ where $N=4$ using DIF FFT Algorithm.

Q.9. Determine Direct form I & II for

$$H(z) = \frac{0.28z^2 + 0.319z + 0.04}{0.5z^3 + 0.3z^2 + 0.17z - 0.2}$$

Q.10. Explain IIR Filter design by the Bilinear Transformation?

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Q.1

Ans:- Properties of LTI system

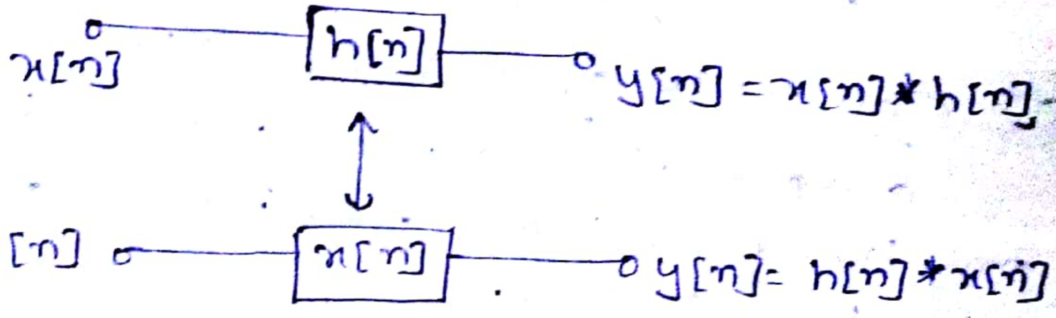
(1) The Commutative Property:-

According to commutative property the output of LTI System is

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

or

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$



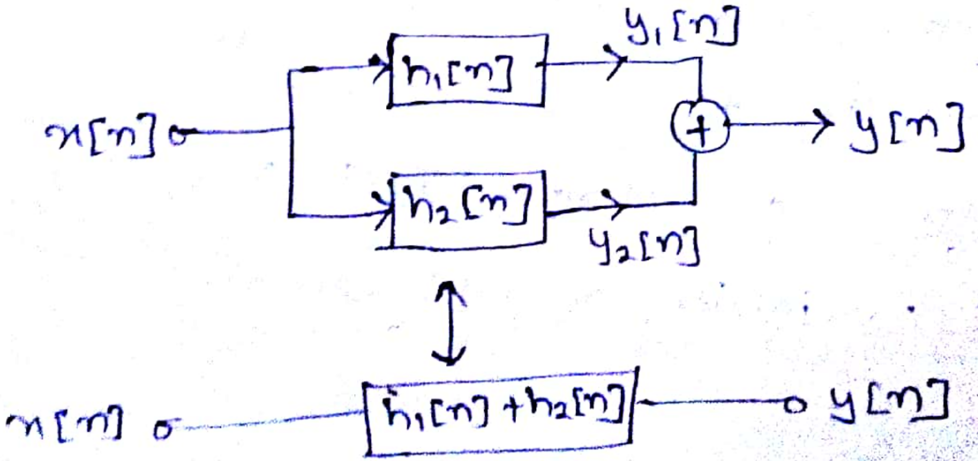
(2) The Distributive Property:-

According to this property

$$y[n] = x[n] * [h_1[n] + h_2[n]]$$

or

$$y[n] = x[n] * h_1[n] + x[n] * h_2[n]$$



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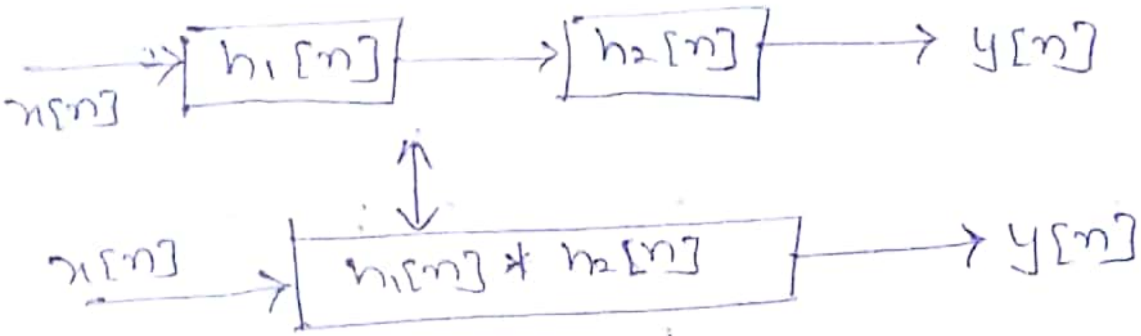
(3) Associative Property of LTI System :-

According to this property

$$y[n] = x[n] * [h_1[n] * h_2[n]]$$

$$y[n] = x[n] * h_1[n] * h_2[n]$$



(4) Causality of LTI System :-

output of a causal systems depends only on the present and past values of the input to the system.

LTI system is causal if its impulse response h[n] is zero for $n<0$.

$$y[n] = x[n] * h[n] = \sum_{k=0}^{\infty} x[k]\, h[n-k]$$

$$y[n] = h[n] * x[n] = \sum_{k=0}^{\infty} h[k]\, x[n-k]$$

(5) Static and Dynamic LTI System :-

(6) Stability of LTI System

(7) Static and Dynamic LTI System

(8) Inveriability of LTI System:

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Q. 2

Ans:- Since $x[n] = 0$ for $n < 0$

& $h[n] = 0$ both are causal.

So

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$= \sum_{k=0}^{n} u[k]\, 2^{n-k} u[n-k]$$

$$y[n] = \sum_{k=0}^{n} 2^{n-k}$$

$$y[n] = 2^{n} \sum_{k=0}^{n} 2^{-k}$$

$$y[n] = 2^{n}\left[1 + 2^{-1} + 2^{-2} + \cdots (n+1) \text{ terms}\right]$$

$$y[n] = 2^{n+1}\left[1 - \left(\tfrac{1}{2}\right)^{n+1}\right]$$

$$y[n] = 2 \cdot 2^{n} - 1 \quad \text{for } n \geq 0$$

or $$y[n] = 2(2)^{n} u[n] - u[n]$$

Q. 3

Ans:- Properties of Z Transform

(1) Linearity:- if $x_1(n) \xleftrightarrow{Z} X_1(z)$

& $x_2(n) \xleftrightarrow{Z} X_2(z)$

then

$$x(n) = a_1 x_1(n) + a_2 x_2(n) \xleftrightarrow{Z} X(z) = a_1 X_1(z) + a_2 X_2(z)$$

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② Time Reversal:-

if $x(n) \xleftrightarrow{Z} X(z)$

then $x(-n) \xleftrightarrow{Z} X(z^{-1})$

③ Time shifting:-

If $x(n) \xleftrightarrow{Z} X(z)$ then

$x(n-k) \xleftrightarrow{Z} z^{-k} X(z)$

④ Scaling:-

If $x(n) \xleftrightarrow{Z} X(z)$ then

$a^n x(n) \xleftrightarrow{Z} X(a^{-1}z)$

⑤ Differentiation:-

If $x(n) \xleftrightarrow{Z} X(z)$ then

$n x(n) \longleftrightarrow -z \dfrac{dX(z)}{dz}$ or $z^{-1} \dfrac{dX(z)}{dz^{-1}}$

⑥ Convolution:-

If $x_1(n) \xleftrightarrow{Z} X_1(z)$

& $x_2(n) \xleftrightarrow{Z} X_2(z)$

then $x(n) = x_1(n) * x_2(n) \xleftrightarrow{Z} X(z) = X_1(z) X_2(z)$

⑦ Initial value Theorem:-

If $x(n)$ is causal system

then $x(0) = \lim\limits_{n \to 0} x(n) = \lim\limits_{|z| \to \infty} X(z)$

⑧: Final value Theorem:-

$x(\infty) = \lim\limits_{n \to \infty} x(n) = \lim\limits_{|z| \to 1} \left[(1-z^{-1}) X(z)\right]$

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Q.4.
Ans: —

$$H(z) = \frac{-4+8z^{-1}}{1+6z^{-1}+8z^{-2}} = \frac{-4+8z^{-1}}{(1+4z^{-1})(1+2z^{-1})}.$$

$$H(z) = \frac{A_1}{1+4z^{-1}} + \frac{A_2}{1+2z^{-1}}$$

$$A_1 = \left.\frac{-4+8z^{-1}}{1+2z^{-1}}\right|_{z^{-1}=-1/4} = \frac{-6}{1/2} = -12$$

$$A_2 = \left.\frac{-4+8z^{-1}}{1+4z^{-1}}\right|_{z^{-1}=-1/2} = \frac{-8}{-1} = 8$$

$$H(z) = \frac{-12}{1+4z^{-1}} + \frac{8}{1+2z^{-1}}$$

Taking inverse Z Transform

$$h(n) = \left[-12(-4)^n + 8(-2)^n\right]u(n)$$

Q.5.
Ans: — Sampling theorem:

For perfect reconstruction of a Continuous-time Signal from its Samples, the Sampling rate or Sampling frequency should be greater than or equal to Nyquist rate of the message signal.

$$\boxed{f_s \geq 2f_m}$$

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Interpolation Techniques:-

It is a procedure for reconstructing a continuous time signal from its sample values.

Let a spectrum of reconstructed signal from its samples $S_r(t)$

$$S_r(t) = S_p(t) * h(t) \quad -(1)$$

$h(t)$ – Impulse response of low pass filter

$S_p(t)$ is given as

$$S_p(t) = \sum_{n=-\infty}^{\infty} s(nT_s)\,\delta(t-nT_s)$$

from (1)

$$S_r(t) = \sum_{n=-\infty}^{\infty} s(nT_s)\,\delta(\cancel{t}-nT_s) * h(t)$$

$$S_r(t) = \sum_{n=-\infty}^{\infty} s(nT_s)\,h(t-nT_s) \quad -(2)$$

Impulse response of low pass filter is given by

$$h(t) = \omega_c T_s \frac{\sin(\omega_c t)}{\pi\,\omega_c T_s}$$

So from (2)

$$S_r(t) = \sum_{n=-\infty}^{\infty} s(nT_s)\left\{\frac{\omega_c T_s}{\pi}\left[\frac{\sin[\omega_c(t-nT_s)]}{\omega_c(t-nT_s)}\right]\right\}$$

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Q. 6

Ans:- Aliasing phenomenon is caused under Sampling. It occurs when sampling rate $[\omega_s < 2\omega_{max}]$.

A phenomenon of high frequency components in a spectrum of a continuous time signal seemingly taking on the identity of a lower freq. in the spectrum of its sampled version

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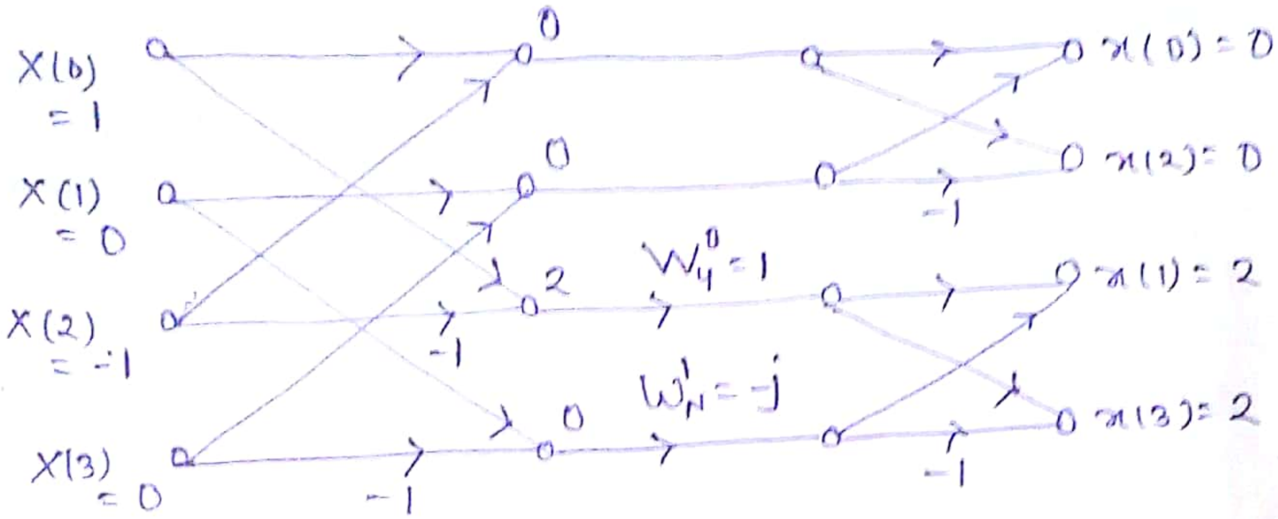
Q. 8.

Ans :- given $N = 4$ and $x(n) = \{1, 0, -1, 0\}$

$$W_N^k = e^{-j\left(\frac{2\pi}{N}\right)k}$$

$$W_4^0 = 1 \quad \text{and} \quad W_4^1 = e^{-j\pi/2} = -j$$



Q. 7

Ans :- N point DFT

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nK/N} \quad , K = 0, 1, \ldots N-1$$

$$x(n) = \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$$

Therefore $X(K) = \frac{1}{4}\left[1 + e^{-j\omega} + e^{-j2\omega}\right]\Big|_{\omega = \frac{2\pi K}{N}}$

$$= \frac{1}{4} e^{-j\omega}\left[1 + 2\cos\omega\right]$$

$$= \frac{1}{4} e^{-j2\pi K/3}\left[1 + 2\cos\left(\frac{2\pi K}{3}\right)\right]$$

$K = 0, 1, \ldots N-1$

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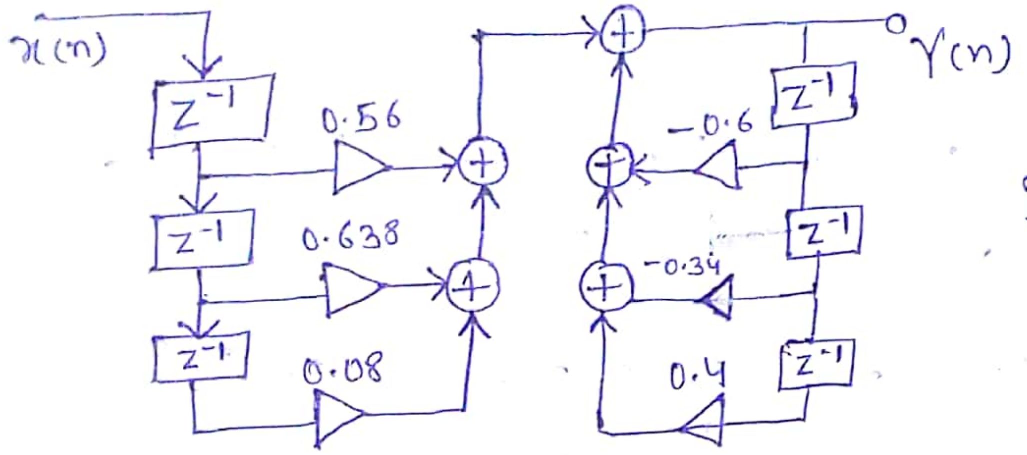
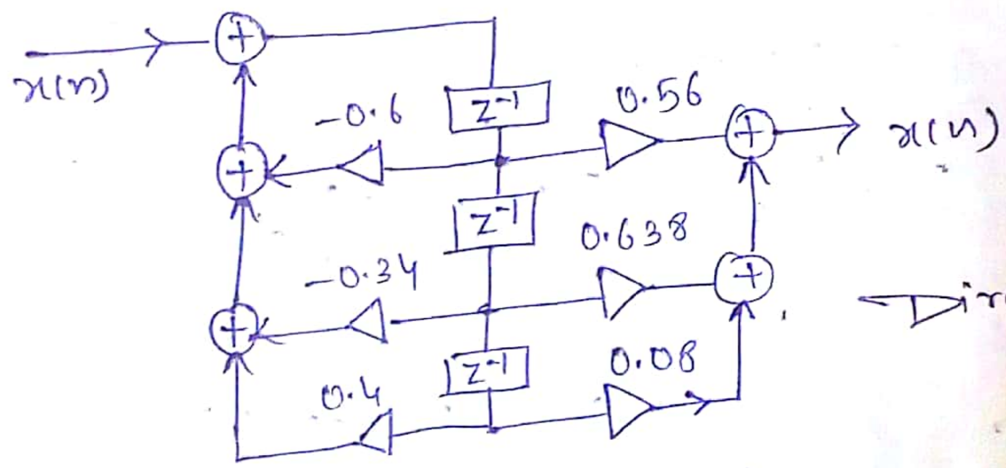
Q.9

Ans:-

$$H(z) = \frac{0.56z^{-1} + 0.638z^{-2} + 0.08z^{-3}}{1 + 0.6z^{-1} + 0.34z^{-2} - 0.4z^{-3}}$$



Direct form I



Direct form II

Q 10.

Ans:- Let the system function of the Analog filter be

$$H(s) = \frac{b}{s+a} \quad -(1)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{b}{s+a}$$

$$sY(s) + aY(s) = +bX(s)$$

Taking inverse Laplace Transform

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$$\frac{dy(t)}{dt} + a\,y(t) = b\,x(t) \quad -(2)$$

By Trapezoidal Rule

$$\int_{nT-T}^{nT} a(t)\,dt = \frac{T}{2}\left[a(nT) + a(nT-T)\right] \quad -(3)$$

integrate Eq^n (2) b/w limits $(nT-T)$ & nT & apply Trapezoidal Rule

$$y(nT) - y(nT-T) + \frac{aT}{2}y(nT) + \frac{aT}{2}y(nT-T)$$

$$= \frac{bT}{2}x(nT) + \frac{bT}{2}x(nT-T)$$

Taking Z Transform

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b}{\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + a} \quad -(4)$$

Compare (1) & (4)

$$S = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) = \frac{2}{T}\left[\frac{z-1}{z+1}\right]$$

By putting S value in any analog filter, can be converted into digital IIR filter